Trig Expressions

Warmup

Find the missing term

1) $\dfrac{1 + \cos x}{1 - \cos x} - \dfrac{1 - \cos x}{1 + \cos x} = 4 \cot x (???)$ $\csc x$

2) $(\sec x - \tan x)^2 = \dfrac{1 - \sin x}{1 + ???}$ $\sin x$

Chapter 6
Solving Trigonometric Equations

1. **Inverse Trigonometric Functions**

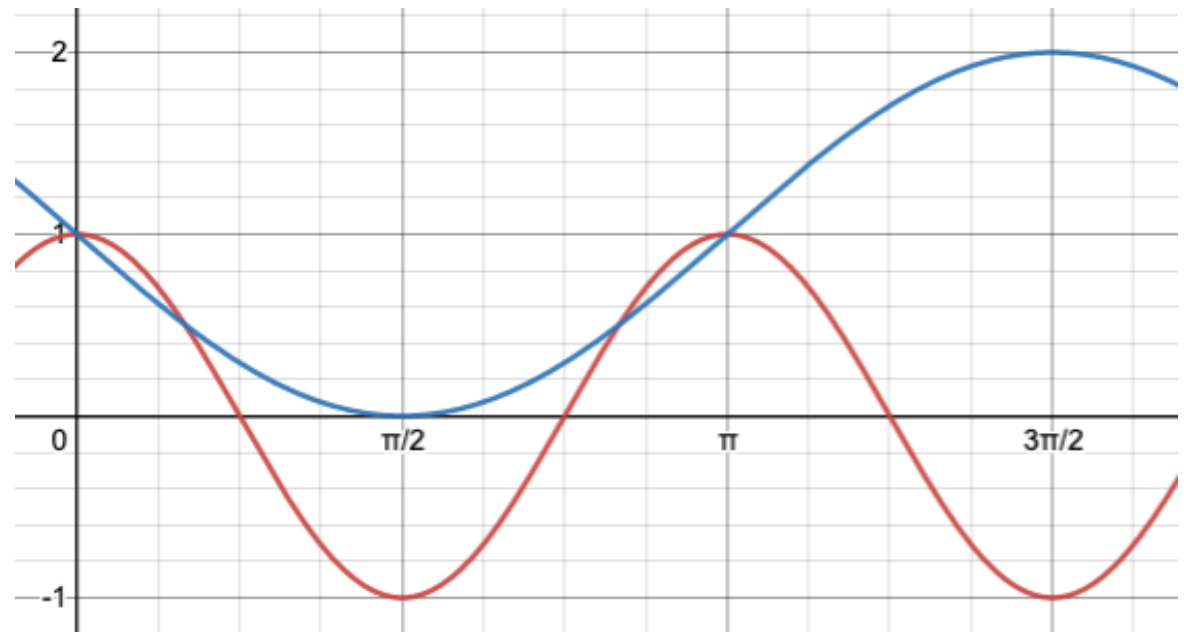2. Solving One Trig Function

3. Solving Multiple Trig Functions

Solving Equations

6.1 - Inverse Trigonometric Functions

Solving equations

$$\cos(2x) = 1 - \sin x$$

$$0 \leq x < 2\pi$$

How many solutions?

4 solutions

$$\cos(2x) = 1 - \sin x$$

$$1 - 2\sin^2(x) = 1 - \sin x$$

$$0 = 2\sin^2(x) - \sin x$$

$$0 = \sin x(2\sin(x) - 1)$$

$$\sin x = 0, \frac{1}{2}$$

$$x = 0, \pi$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

6.1 - Inverse Trigonometric Functions

Solving equations

$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \cos(2A) = 1 - 2\sin^2 A \qquad \tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin(2A) = 2\sin A \cos A \qquad \cos(2A) = 2\cos^2 A - 1$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}} \qquad \tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A} \qquad \tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

Practice - Solve in radians

1) $\sin(2x) = \cos x$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

2) $\sin^2 x = 2\sin^2\left(\frac{x}{2}\right)$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, 0$$

Trig Identities

6.1 - Inverse Trigonometric Functions

Solving equations

$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \cos(2A) = 1 - 2\sin^2 A \qquad \tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin(2A) = 2\sin A \cos A \qquad \cos(2A) = 2\cos^2 A - 1$$

$$\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos\theta}{2}} \qquad \tan\left(\frac{A}{2}\right) = \frac{\sin A}{1 + \cos A} \qquad \tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}$$

Practice - Combine into one term

$$1) \ \frac{2(1 + \cos x)}{\csc^2\left(\frac{x}{2}\right)}$$

$$\sin^2 x$$

Trig Equations

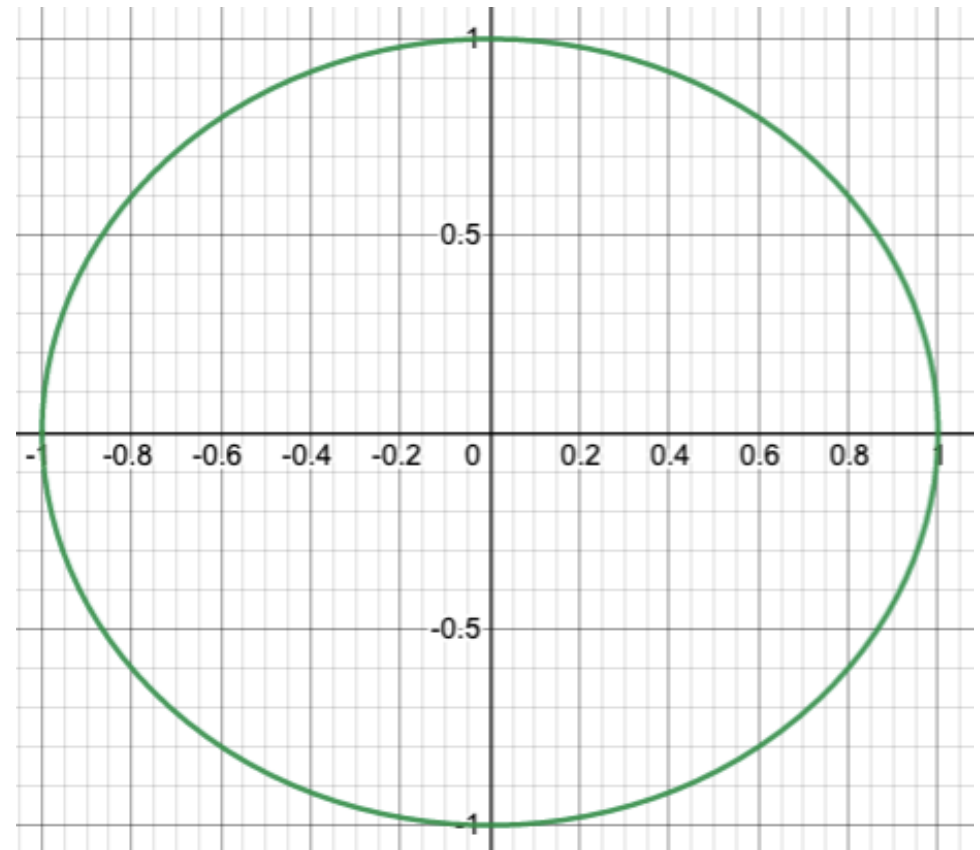# 6.1 - Inverse Trigonometric Functions

Trig equations

$$\sin(x) = 0.6$$

$$0 \le x < 2\pi$$

$$x = 36.9° = 0.64 \; rads$$

$$x = 143.1° = 2.50 \; rads$$

What about cos and tan?

6.1 - Inverse Trigonometric Functions

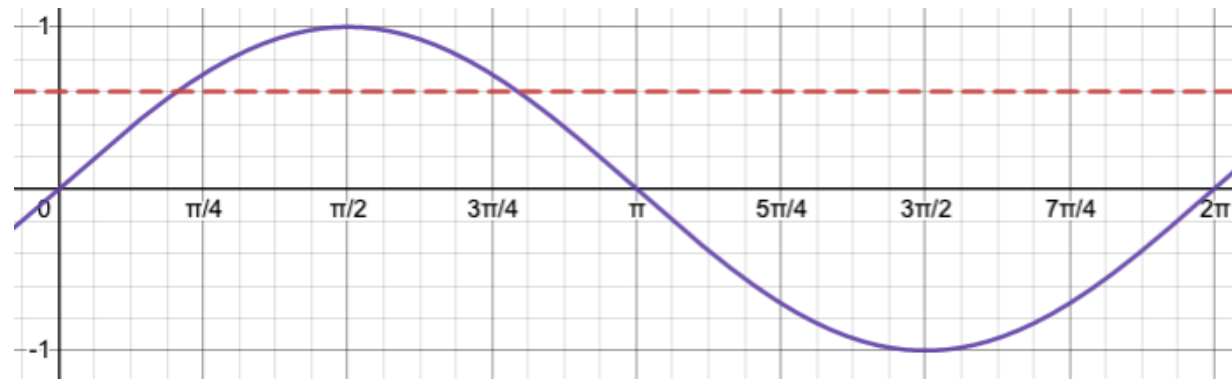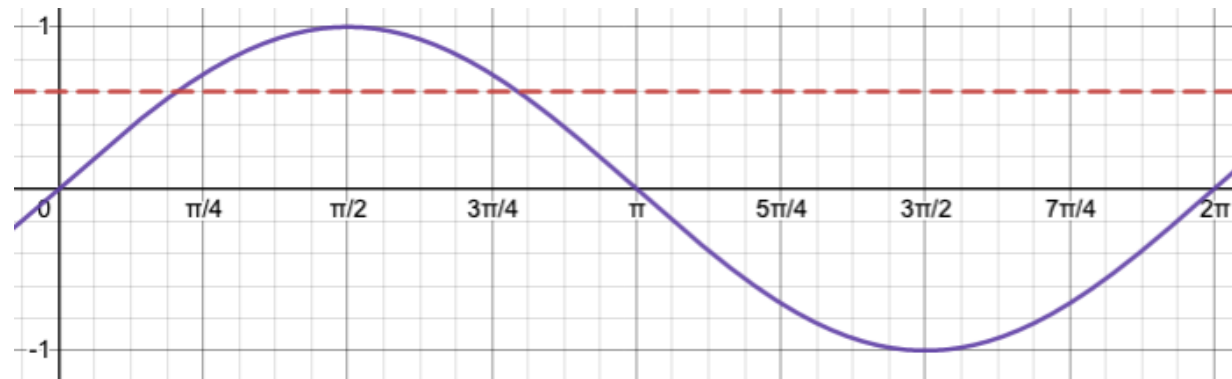**Trig equations**

$$\sin(x) = 0.6$$
$$0 \le x < 2\pi$$

$$x = 36.9° = 0.64 \; rads$$

$$x = 143.1° = 2.50 \; rads$$

Practice - Leave answer in radians $\quad 0 \le x < 2\pi$

1) $\sin x = -0.2$

2) $\cos x = 0.7$

3) $\sec x = 2.2$

$x = 6.08, 3.34$ $\qquad$ $x = 0.80, 5.49$ $\qquad$ $x = 1.10, 5.18$

6.1 - Inverse Trigonometric Functions

Trig equations

$\sin(x) = 0.6$

$\quad 0 \leq x < 2\pi$

$x = 36.9° = 0.64 \ rads$

$x = 143.1° = 2.50 \ rads$

What about $\csc(x) = 1.667$?

$\dfrac{1}{\sin x} = 1.667$

$x = 36.9° = 0.64 \ rads$

$x = 143.1° = 2.50 \ rads$

$\sin x = \dfrac{1}{1.667} = 0.6$

Inverse Trig Functions

6.1 - Inverse Trigonometric Functions

Warmup

Find the exact value for each expression in radians.

1. $tan^{-1}\left(-\sqrt{3}\right)$

$$-\frac{\pi}{3}$$

2. $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$$\frac{5\pi}{6}$$
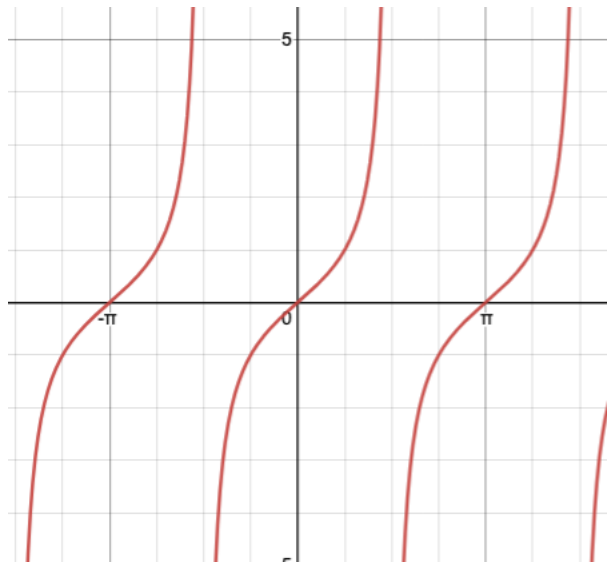
3. $sin^{-1}\left(-\frac{1}{2}\right)$

$$-\frac{\pi}{6}$$

4. $csc^{-1}\left(\frac{2\sqrt{3}}{3}\right)$

$$\frac{\pi}{3}$$
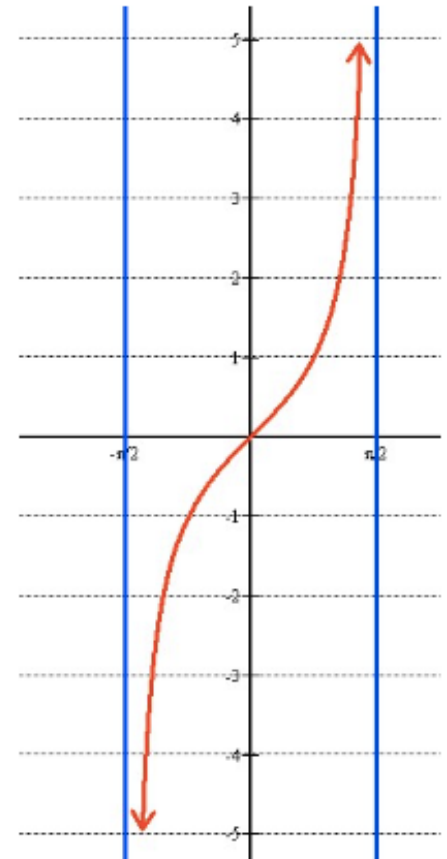
6.1 - Inverse Trigonometric Functions
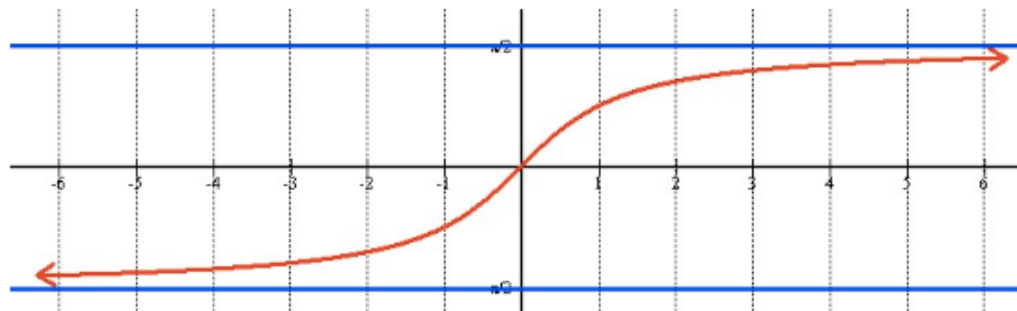
Inverse

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$f(\theta) = \tan \theta$

$F(\theta) = Tan\theta$

$$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

Is this a function?
Is the inverse a function?

Is this a function?

Is the inverse a function?
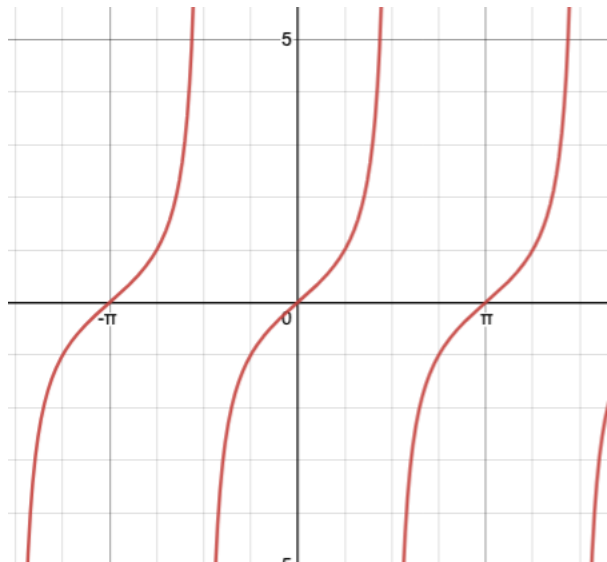
$$F^{-1}(\theta) = Tan^{-1}\theta$$
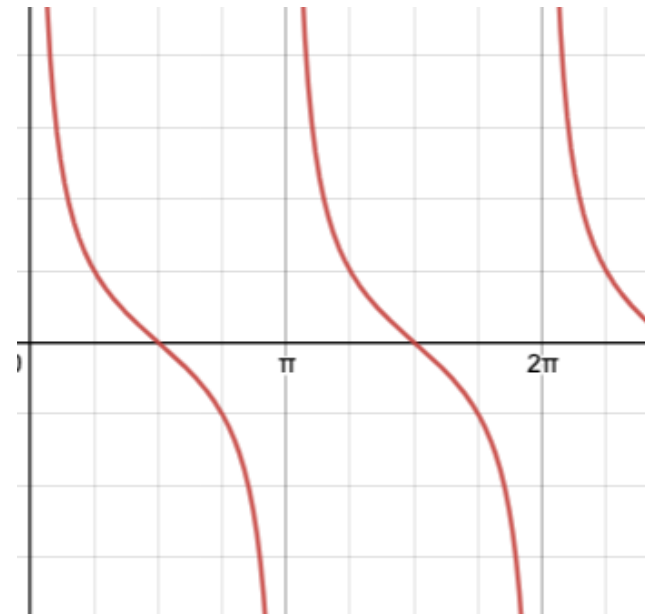
6.1 - Inverse Trigonometric Functions

Inverse

$$f(\theta) = \tan\theta$$

Is this a function?
Is the inverse a function?

What about Cot?

$$0 < \theta < \pi$$

6.1 - Inverse Trigonometric Functions

Inverse

Find $Tan^{-1}(2)$ $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

Use your calculator

$Tan^{-1}(2) = 63.4° = 1.1 \ rads$

Find $Tan^{-1}(-1)$

$Tan^{-1}(-1) = -45° = -\dfrac{\pi}{4}$

Is -1 a degree or rad?

6.1 - Inverse Trigonometric Functions

Inverse

Find $Tan^{-1}(2)$ $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$ Find $Tan^{-1}(-1)$

Use your calculator

$$Tan^{-1}(-1) = -45° = -\dfrac{\pi}{4}$$

$$Tan^{-1}(2) = 63.4° = 1.1 \ rads$$

Is -1 a degree or rad?

Practice - Leave answer in radians

1) $Tan^{-1}\left(\dfrac{7}{3}\right)$ 2) $Tan^{-1}\left(-\dfrac{7}{3}\right)$ 3) $Cot^{-1}\left(\dfrac{5}{6}\right)$

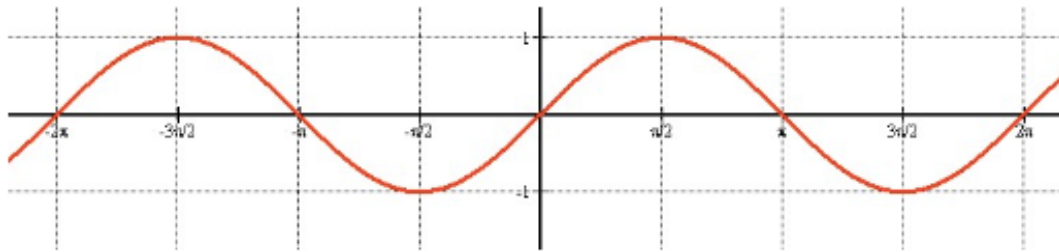 $1.2 \ rads$ $-1.2 \ rads$ $0.9 \ rads$

6.1 - Inverse Trigonometric Functions

Inverse

$$f(\theta) = \sin\theta$$

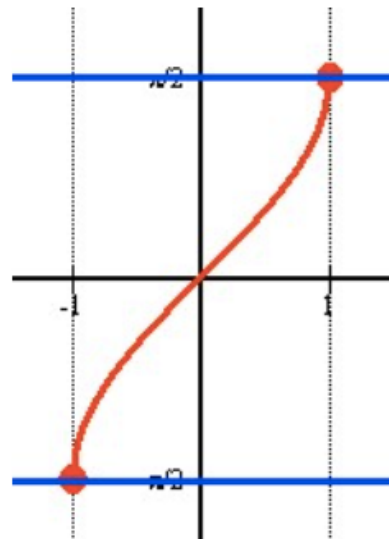$$F(\theta) = Sin\theta$$

$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

Is this a function?
Is the inverse a function?
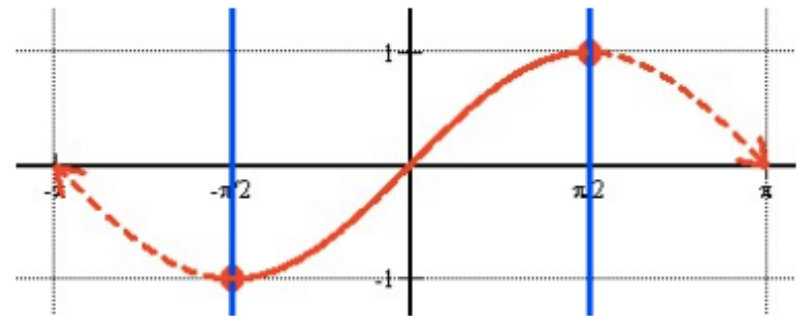
Is this a function?
Is the inverse a function?

$$F^{-1}(\theta) = Sin^{-1}\theta$$

6.1 - Inverse Trigonometric Functions
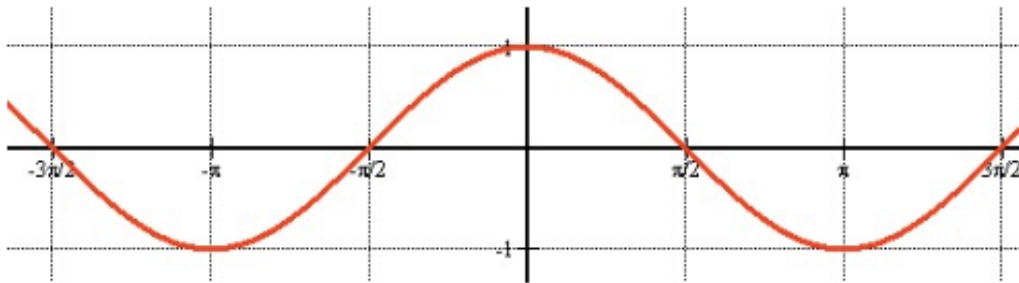
Inverse

$$f(\theta) = \cos \theta$$

$$F(\theta) = Cos\theta$$

$$0 \le \theta \le \pi$$

Is this a function?
Is the inverse a function?

Is this a function?
Is the inverse a function?

$$F^{-1}(\theta) = Cos^{-1}\theta$$

6.1 - Inverse Trigonometric Functions

Inverse
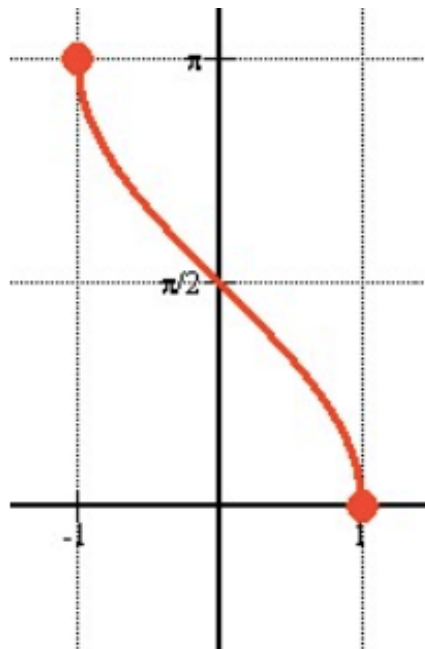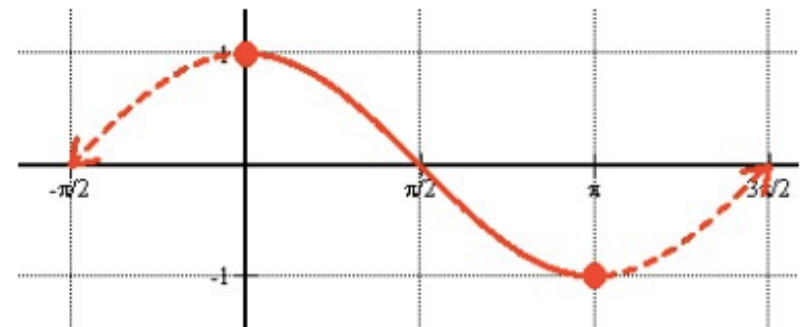
$$F(\theta) = Tan\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$F(\theta) = Sin\theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$F(\theta) = Cot\theta \quad 0 < \theta < \pi$$

$$F(\theta) = Cos\theta \quad 0 \leq \theta \leq \pi$$

Practice - Leave answer in radians

1) $Sin^{-1}(-0.8)$

2) $Cos^{-1}(-0.5)$

3) $\cos\left(Cos^{-1}\frac{5}{6}\right)$

$-0.93 \; rads$

$2.1 \; rads$

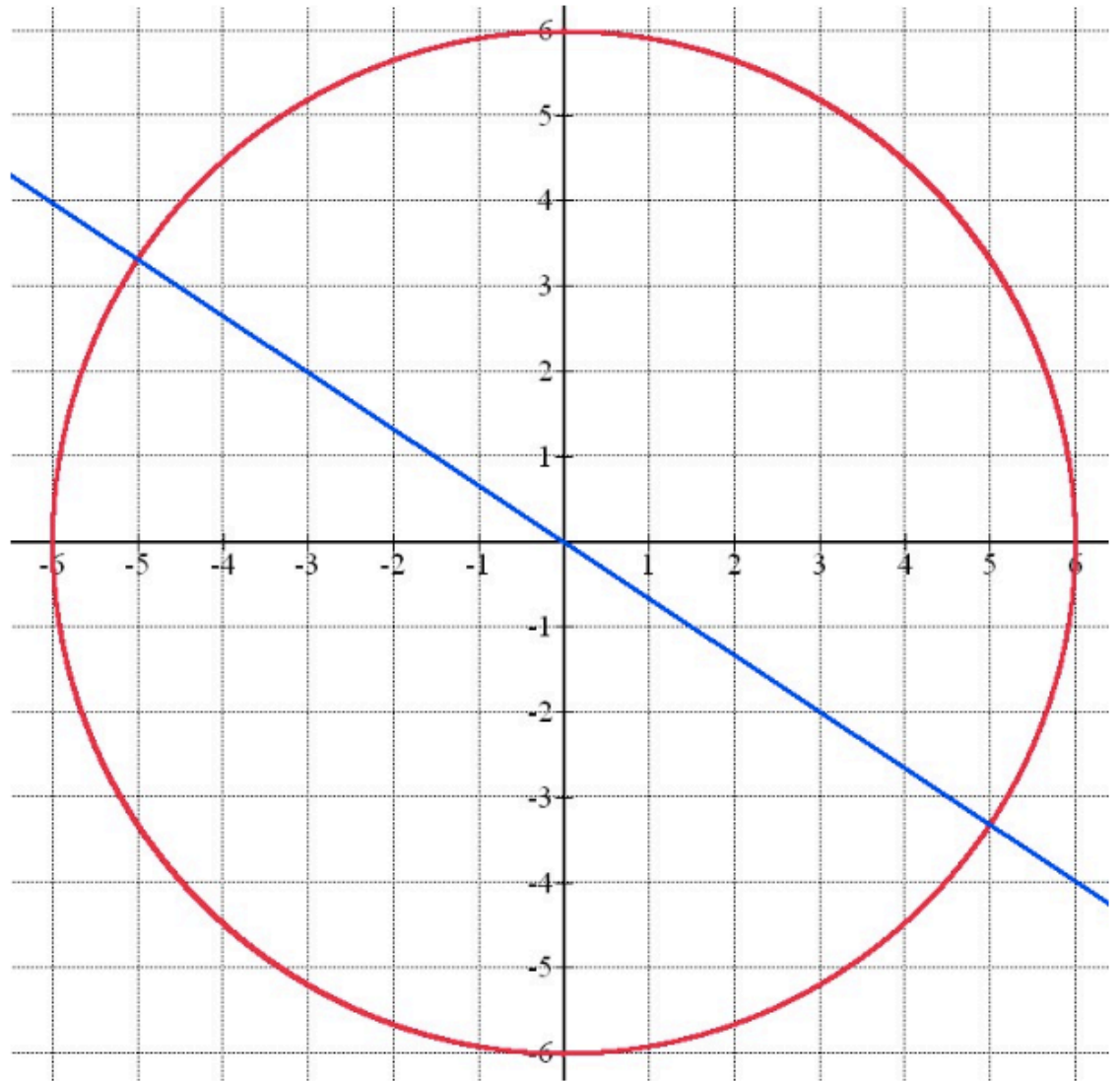$\frac{5}{6}$

6.1 - Inverse Trigonometric Functions

Inverse

$$\sin \left(Cos^{-1} -\frac{5}{6} \right)$$

Solve without a calculator

$$Cos^{-1} -\frac{5}{6} = \theta$$

$$\sin \theta = \frac{\sqrt{11}}{6}$$

6.1 - Inverse Trigonometric Functions

Inverse

$$F(\theta) = Tan\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \qquad\qquad F(\theta) = Sin\theta \quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$F(\theta) = Cot\theta \quad 0 < \theta < \pi \qquad\qquad F(\theta) = Cos\theta \quad 0 \le \theta \le \pi$$

Practice - Leave answer in fractions. Solve without calculator

1) $\tan\left(Cos^{-1}\left(-\frac{12}{13}\right)\right)$ 2) $\sin\left(Cos^{-1}\left(-\frac{1}{5}\right)\right)$ 3) $\sec\left(Sin^{-1} x\right)$

$$-\frac{5}{12} \qquad\qquad \frac{2\sqrt{6}}{5} \qquad\qquad \frac{\sqrt{1-x^2}}{1-x^2}$$

6.1 - Inverse Trigonometric Functions

Inverse

$$f(x) = \cos(\cos^{-1} x)$$

What is the domain?

What is the range?

$$-1 \leq x \leq 1$$

$$-1 \leq y \leq 1$$

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